Theoretical and Empirical Studies of the Role of Modified Integration Kernels in Gravimetric Geoid Determination

Degree:	PhD
Keywords:	geodesy, geoid, gravity, modified kernels, optimised filters, truncation error reduction, satellite gravity gradiometry, approximation theory
Entry:	Bachelors, preferably 1st class Honours, Postgraduate Diploma or Masters degree in geoscience, physics, mathematics, or any related discipline
Supervisor:	Professor Will Featherstone (W.Featherstone@curtin.edu.au), Western Australian Centre for Geodesy, Department of Spatial Sciences, Curtin University of Technology
Project Funding:	Australian Research Council, Western Australian Centre for Geodesy
Student Funding:	Department of Spatial Sciences Scholarship, or other scholarships offered by the University (http://www.scholarships.curtin.edu.au/)
Resources:	Australian gravity and terrain data, global geopotential models, some kernel modification software, Australian GPS and AHD data
Collaboration:	Geoscience Australia, University of New Brunswick (Canada), and Royal Institute of Technology (Sweden)
Starting Date:	Unrestricted

Project Description:

Following the pioneering work of Molodensky (Molodensky *et al*., 1962), modified integration kernels are receiving some renewed increased attention in regional gravimetric geoid determination. A notable example is the determination of AUSGeoid98, where a new modified kernel (Featherstone *et al*., 1998) was used. However, there are a variety of other kernel modifications available (including and cited in Evans and Featherstone, 2000 and Sjöberg and Hunegnaw, 2000). Importantly, each kernel appears to behave differently in different parts of the world, depending on the error characteristics of the data in each region (eg. Forsberg and Featherstone, 1998). However, not all kernels have been compared under the same conditions so as to give an objective analysis.

In the geodetic literature, there appear to be two schools of thought concerning the use of modified kernels in gravimetric geoid determination, with some using a rectangular gravity data area with an unmodified form of Stokes's kernel and others using a spherical cap in conjunction with a (deterministically or stochastically) modified integration kernel. The preferential choice between these has not yet been resolved. There are also many different modified kernels, most of which have been formulated predominantly to reduce the truncation error that occurs when using data over a limited area (see the citations in Featherstone *et al*., 1998, Vanicek and Featherstone, 1998 and Jekeli, 1981). To date, however, it is unclear which is the optimal approach, with different kernels appearing to give better results in different parts of the world (eg. Forsberg and Featherstone, 1998).

The proposed project will involve research into one or more of the following:

- Review all of the existing kernel modifications and place them into a self-consistent framework, identifying the advantages and disadvantages of each (cf. Vanicek and Featherstone, 1998). Essentially, there are two classes of kernel modification: deterministic and stochastic, with some commonality between them. Such a review of the existing modified kernels will probably identify new classes of kernel modification that exploit the benefits of each. Indeed, it is conceivable that a combination of several modifications can be used in different frequency bands to gain an optimal combined solution for the geoid. Featherstone (2002) has proposed such an approach, albeit in an embryonic form.

- Given that each kernel behaves differently in different parts of the world (eg. Forsberg and Featherstone, 1998), empirical tests will be conducted using observed gravity and terrain data and the results compared to Global Positioning System (GPS) and levelling data on the local vertical datum (Featherstone and Sideris, 1998). A set of 1013 GPS and AHD data is available in Australia from the project supervisor. However, such tests are subject to many other errors. Therefore, a synthetic gravity field, where the exact relationship between gravity anomalies and geoid heights is known, will be used to test the kernel modifications (cf. Featherstone, 1999). This will be extended with the introduction of synthetic errors to determine how the modified kernels behave in the presence of these errors.

- Modified kernels are known to behave as filters (eg. Vanicek and Featherstone, 1998). Therefore, they can be combined and optimised to gain the best estimate of the geoid, knowing the data errors. Both global and local models of these errors will be used to determine the accuracy of the computed geoid. For instance, satellite-derived gravity data provide a better source of long-wavelength gravity field information than terrestrial gravity observations, so the terrestrial data should be filtered as much as possible. The reliance on the satellite-derived component of the geoid will increase in future with the planned dedicated satellite gravity field missions (eg GRACE, CHAMP and GOCE). Therefore, the filtering properties of the modified kernels will also be optimised with a view to improved geoid solutions using satellite gravity data, as well as airborne gravity data.

Recommended Reading:

Evans, J.D. and W.E. Featherstone (2001) Improved convergence rates for the truncation error in gravimetric geoid determination. *Journal of Geodesy*, 74: 239-248.

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Vanicek, P. and W.E. Featherstone (1998) Performance of three types of Stokes's kernel in the combined solution for the geoid, *Journal of Geodesy*, 72(12): 684-697.

Wenzel H-G (1982) Geoid computation by least squares spectral combination using integral kernels, *Proc IAG General Meeting*, Tokyo, 438-453.

Wong L, Gore R (1969) Accuracy of geoid heights from modified Stokes kernels, *Geophysical Journal of the Royal Astronomical Society*, 18: 81-91.

Zelin, G. and Zuofa, L. (1992) Modified Stokes’s integral formulas using FFT. *manuscripta geodaetica*, 17: 227-232.