# A Comparison of Mathematical Models and Numerical Accuracy of Gravimetric Terrain Corrections

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| **Degree:** | PhD |
| **Key-words:** | geophysics, geodesy, gravity field, terrain corrections, error propagation, topography, digital elevation models |
| **Entry:** | Bachelors, preferably 1st class Honours, Postgraduate Diploma or Masters degree in geoscience, physics, mathematics, or any related discipline |
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| **Project Funding:** | Australian Research Council |
| **Student Funding:** | Department of Spatial Sciences Scholarship |
| **Resources:** | Australian gravity and terrain data, terrain correction computation software via FFT, LSC and quadrature methods |
| **Collaboration:** | Geoscience Australia, International Association of Geodesy, Society for Exploration Geophysics |
| **Starting Date:** | Unrestricted |

**Project Description:**

The terrain correction is essential in both the disciplines of geophysics and geodesy. In geophysics, the terrain correction (Bullard C correction) is used to model and *remove* the gravitational effects of the topography residual to the Bouguer plate/cap/shell. Of course, the terrain correction used must be consistent with the spherical or planar Bouguer model (cf. LaFehr, 1991; Takin and Talwani, 1996; Vanicek *et al*., 2001). In geodesy, the terrain correction is used as part of a 'condensation' reduction (normally according to Helmert's second method) to replace the gravitational effect of the *in situ* topographic masses with an equivalent layer situated (condensed/compressed) at the geoid (eg., Martinec and Vanicek, 1994).

Over the decades, a plethora of different mathematical models have been proposed for the terrain correction, many of which are cited in the recommended reading list below. However, there is no clear consensus either within each discipline or across disciplines on how to optimally compute the terrain correction to gravity data. This project aims to rectify this.

The proposed project will involve research into one or more of the following:

- Review all of the existing mathematical models for the terrain correction and place them into a self-consistent framework, identifying the advantages and disadvantages of each. Such a review of the existing algorithms will probably identify new classes of terrain correction model that exploit the benefits of the existing models. This is necessary because there appears to be much ambiguity, confusion and debate over the application of the ‘terrain correction’ (cf. Hammer, 1982), especially as each discipline appears to use the same terminology to describe slightly different processes. Moreover, some of the terrain correction models used in geodesy are equivalent to the geophysical terrain correction under certain assumptions (eg. Moritz, 1968; Li and Sideris, 1994). Importantly, the study will source all the literature from each discipline. This should help to further clarify the differences/ambiguities between the geodetic terrain correction and the geophysical terrain correction, if indeed there is one.

- Normally the terrain correction is evaluated out to a certain distance from the computation point. Therefore, this distance should be optimised. However, several mathematical models have been proposed to consider the remote zone effects of the terrain correction over the whole Earth (eg Novak *et al*., 2001; Danes, 1982; Smith, 2001), which appear to be significant when using spherical models of the global terrain effect. Therefore, these different models should be compared and contrasted in their own right. Moreover, they should be evaluated when taking into account the accuracy and availability of global digital elevation models.

- There is also a disparity between the geophysical and geodetic applications of the terrain correction. Geophysicists require that the terrain correction be applied to the gravity observation point. However, it is far more convenient to compute the terrain corrections from an existing regular digital elevation model. This raises the problem of how to incorporate the near-meter effects (cf. Leaman, 1998; Nowell, 1999), while also taking into account the numerical instabilities due to some planar approximations of the terrain correction (eg. Moritz, 1968; Martinec *et al*., 1996; Kirby and Featherstone, 1999) and weak singularities in the terrain correction kernel (eg. Klose and Ilk, 1993).

- As well as global terrain corrections and terrain corrections based on a DEM, it is also useful to consider the use of more realistic models of the topographic morphology than the flat-topped prisms given in a DEM (eg. Ma and Watts, 1984; Blais and Ferland, 1984, Cogbill, 1990; Barrows and Fett, 1991; Smith, 2000). Again, these different mathematical models should be evaluated when taking into account the accuracy and availability of global digital elevation models.

- The terrain correction is also a function of the topographic mass-density. However, many previous studies (especially those on the geometry of the mathematical models used) tend to omit lateral and vertical variations in the topographic mass density. Therefore, in parallel with or subsequently to the above evaluations, the role of topographic mass-density must be considered. A less significant consideration is the role of the newly estimated value for the universal gravitational constant in the computation of terrain corrections to gravity data.

**Recommended Reading:**


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