

# Curtin Synthetic Earth Gravity Model (Version 1)

## Model Description



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[http:// www.cage.curtin.edu.au/~kuhnm/CurtinSEGM/index.html](http://www.cage.curtin.edu.au/~kuhnm/CurtinSEGM/index.html)



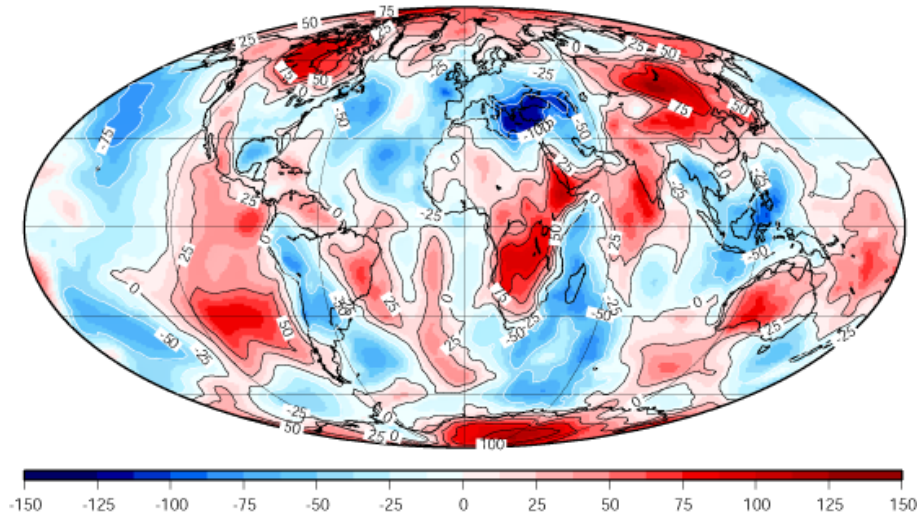
## Introduction

The Curtin Synthetic Earth Gravity Model (CurtinSEGM) describes as best as possible (using current data) the Earth's external gravity field by a set of fully normalised spherical harmonic coefficients of the disturbing potential and the corresponding geoid height. The CurtinSEGM is based on forward gravity field modelling only using mass-density information of topography, bathymetry, crust and mantle. This document provides a brief overview of the data and techniques used to construct CurtinSEGM. Further information can be found in Kuhn and Featherstone (2003a and b). However, Version 1 of CurtinSEGM (presented here) provides slightly different numerical results with respect to the preliminary model described in Kuhn and Featherstone (2003a and b).

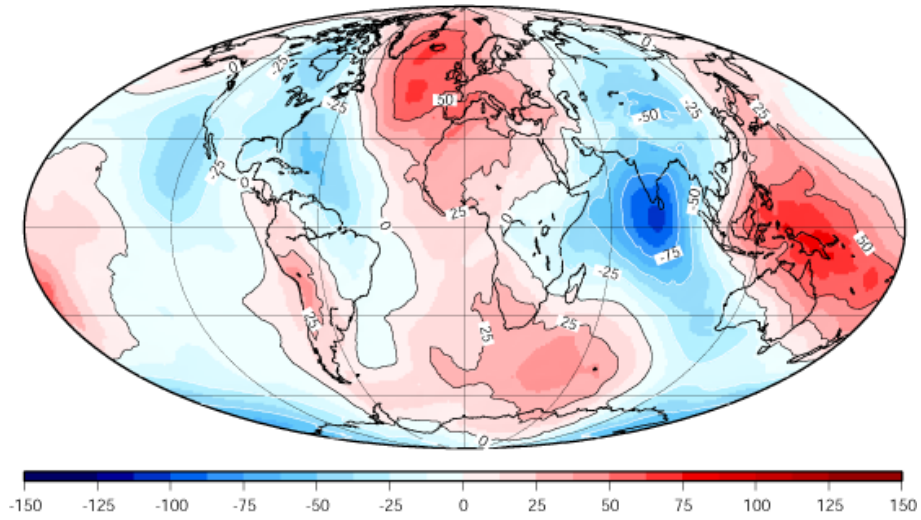
Although CurtinSEGM is a synthetic (simulated) model, the aim was to be as realistic as possible. Figure 1 and 2 show the geoid height of the current model as well as that of the EGM96 (Lemoine et al. 1998), respectively. It can be seen that CurtinSEGM is able to describe the general structure of the anomalous Earth's gravity field. However, in some areas the differences in geoid height with respect to EGM96 almost reach the value of the signal itself (Figure 3). A possible cause can be the lack of information in the mantle for degrees greater than N=12. An overview of selected statistical parameters of CurtinSEGM, EGM96 and their differences are given in Table 1.

**Table 1:** Descriptive statistics for the geoid heights of CurtinSEGM and EGM96 based on a global 1-degree x 1-degree grid derived from spherical harmonics up to degree and order N=360.

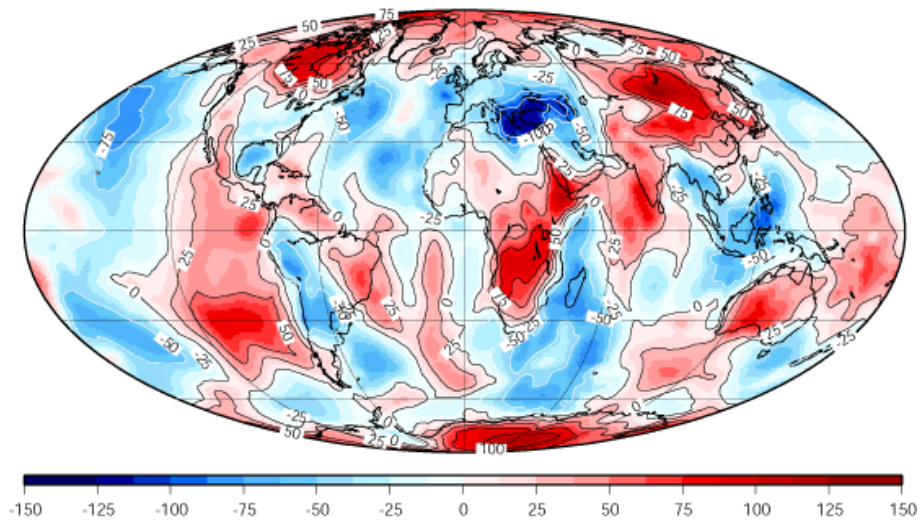
	<b>CurtinSEGM</b>	<b>EGM96</b>	<b>Difference</b>
Mean [m]	8.7	-8.1	9.6
Min [m]	-126.3	-105.8	-128.8
Max [m]	+138.3	+85.2	+138.2
Stdv. [m]	47.5	29.2	43.8



**Figure 1:** Geoid height as given by CurtinSEGM [m]



**Figure 2:** Geoid height as given by EGM96 [m]



**Figure 3:** Difference in geoid height (CurtinSEGM – EGM96) [m]

## Data: Synthetic Earth Mass Model

The construction of CurtinSEGM is based on mass-density information of topography, bathymetry, crust and mantle. An overview of the data (models) used is given in Table 2. Each part of the mass-density information used will be briefly described including the adopted reference density model.

**Table 2:** Data (models) used to construct CurtinSEGM

Description	Model
Topography/Bathymetry	JGP95E (Lemoine et al. 1998)
Crust	CRUST 2.0 (Mooney et al. 1998)
Mantle	S12WM13 (Su et al. 1994)

### **Topography and Bathymetry (JGEP95E)**

Topographic heights and bathymetric depths as well as information on oceanic and glacier ice and lake water are taken from the global 5-min x 5-min JGP95E digital elevation model (DEM; Lemoine et al. 1998, chap. 2). The different terrain types have been converted into equivalent rock heights (rigorous spherical approximation) using  $\rho = 2670 \text{ kg/m}^3$  as reference density (e.g. Kuhn and Seitz 2003).

### **Crust (CRUST 2.0)**

Crustal mass anomalies are taken from the global 2-deg x 2-deg CRUST 2.0 model, which (oddly) is an update of the CRUST 5.1 model (Mooney et al. 1998). In this model, the crust is defined by the depth and laterally variable mass-density of five different layers (soft and hard sediments, upper, middle and lower crust). The bottom of the lowest layer defines the Mohorovičić discontinuity (Moho) used to model the topography of the crust/mantle transition zone.

### **Mantle (S12WM13)**

Density heterogeneities throughout the whole mantle (between 25 km and 2891 km depth) have been derived from the 3-D seismic shear-wave velocity heterogeneity model S12WM13 (Su et al. 1994). This model is expanded to spherical harmonic degree 12, to describe the horizontal variation, and to order 13 in Chebyshev polynomials to describe radial variations. The seismic velocity heterogeneity  $\delta v/v_0$  has been expressed with respect to the preliminary reference Earth model (PREM, Dziewonski and Anderson 1981). Here  $\delta v/v_0$  has been extracted every 25 km in depth between 25 km and 2875 km, expressed by their fully normalised spherical harmonic coefficients  $\delta v_{nm,i}$  referring to the mean sphere of radius  $R_i = R - i \cdot 25 \text{ km}$  [ $i = 1, \dots, 115$ ] with the mean Earth radius  $R=6378137 \text{ m}$ . The set of all seismic velocity heterogeneities ( $\delta v_{nm,i}$ ) has been converted to mass-density heterogeneities (surface density) via

$$\bar{\kappa}_{1,nm,i} = \Delta r \frac{\rho_{0,i}}{\delta v / \delta \rho} \bar{\delta v}_{nm,i},$$

where the constant factor  $\delta v / \delta \rho = 10$ , which relates velocity changes to mass-density changes, has been used. The reference density  $\rho_{0,i}$  for each depth is that of PREM and  $\Delta r$  indicates the vertical extension of each layer (here  $\Delta r = 25 \text{ km}$ ).

## Reference Density Model

The reference density model is assumed to be spherical with concentric layers of constant density. Within the crust of 25 km thickness, the reference crustal density is  $\rho_c = 2861 \text{ kg/m}^3$ , which is the mean density of CRUST 2.0 derived from the five crustal layers. Beneath the reference crust (first 25 km of depth) the density distribution of the PREM has been adopted. The mean density of masses below the core mantle boundary (CMB) then has to be chosen such that the reference model represents the remaining mass of the Earth. Therefore the core has been assumed to have a constant density so that the total mass of the Earth ( $M=5.9737 \cdot 10^{24} \text{ kg}$ ) remains unchanged. An overview of all parameters used to construct the CurtinSEGM are given in Table 3.

**Table 3:** Overview of all used and reference parameters in CurtinSEGM.

Model parameter	Reference Model
<b>General (adapted from GRS80)</b>	
The used parameter are that of the reference model (GRS80) (Moritz 1980)	<p><i>Original Parameter:</i>  <math>R = 6378137.0 \text{ m}</math> Semimajor axis (Here: Mean Earth radius)  <math>GM = 3986005 \cdot 10^8 \text{ m}^3/\text{s}^2</math> Geocentric gravitational constant  <math>J_2 = 108263 \cdot 10^{-8}</math> dynamic factor (not used)  <math>\omega = 7292115 \cdot 10^{-11} \text{ rad/s}</math> mean angular velocity (not used)</p> <p><i>Derived Parameter:</i>  <math>\gamma = 9.797645 \text{ m/s}^2</math> Mean gravity value (average over ellipsoid)  <math>M = 5.973700 \cdot 10^{24} \text{ kg}</math> Mean Earth mass with G given by  <math>G = 6.67259 \cdot 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2</math> Newtonian constant of gravitation</p>
<b>Topography/Bathymetry (JGP95E)</b>	
$2670 \text{ kg/m}^3$ Mean density of topography $1036 \text{ kg/m}^3$ Mean density of ocean water $1000 \text{ kg/m}^3$ Mean density of lake water $927 \text{ kg/m}^3$ Mean density of ice	$2200 \text{ kg/m}^3$ Reference density for equivalent rock heights (Topography, $H^{\text{eth}} > 0$ ) $2670 \text{ kg/m}^3$ Reference density for equivalent rock heights (Bathymetry, $H^{\text{eth}} > 0$ )
<b>Crust (CRUST 2.0)</b>	
<p><i>Parameter adapted from CRUST 2.0</i></p> $1998.01 \text{ kg/m}^3$ Mean density of soft sediment layer $2346.62 \text{ kg/m}^3$ Mean density of hard sediment layer $2675.42 \text{ kg/m}^3$ Mean density of upper crust layer $2888.94 \text{ kg/m}^3$ Mean density of middle crust layer $3044.53 \text{ kg/m}^3$ Mean density of lower crust layer $3365.38 \text{ kg/m}^3$ Mean density of upper mantle $2406.98 \text{ m}$ Mean depth of soft sediment layer $2967.36 \text{ m}$ Mean depth of hard sediment layer $6583.09 \text{ m}$ Mean depth of upper crust layer $13314.68 \text{ m}$ Mean depth of middle crust layer $19839.12 \text{ m}$ Mean depth of lower crust layer $22974.75 \text{ m}$ Mean crustal thickness (MOHO)	$2200 \text{ kg/m}^3$ Density of topography ( $\rho^t$ ) $2861 \text{ kg/m}^3$ Density of crust ( $\rho^c$ ) $3381 \text{ kg/m}^3$ Density of upper mantle ( $\rho^{\text{m}25}$ ) [25 – 50 Km] $3378 \text{ kg/m}^3$ Density of upper mantle ( $\rho^{\text{m}50}$ ) [50 – 75 Km] $25000 \text{ m}$ Crustal thickness (T) <p><i>Density anomalies:</i>  <math>\rho - \rho^t</math> above MSL  <math>\rho - \rho^c</math> crust [0 – 25 Km]  <math>0.77 (\rho - \rho^{\text{m}25})</math> upper mantle [25 – 50 Km]  <math>0.77 (\rho - \rho^{\text{m}50})</math> upper mantle [50 – 75 Km]</p> <p><math>\rho</math> is the lateral variable density within each crustal layer</p>
<b>Mantle (S12WM13)</b>	
Lateral variable density anomalies have been extracted from S12WM13 for 115 mantle layers between 25 Km and 2891 Km depth. The contribution of each layer has been adjusted by the choice of a set of Love numbers (see methodology). No special modelling has been made for the first two layer [25 – 75 Km] as crustal masses partly reach in these layer	Reference velocity and reference density for each mantle layer has been taken from PREM.
<b>Core</b>	
Has not been included.	The mean density of masses below the core mantle boundary (CMB) has to be chosen such that the reference model represents the remaining mass of the Earth.

## Methodology: Disturbing Potential and Geoid Height

The (spherical) reference density model used (see above) is assumed to cause the gravitational part of the reference gravity model (atmosphere has been neglected). Assuming the same centrifugal potential for the reference model and the SEGM the disturbing potential of the SEGM is directly given by the gravitational potential effect of all mass-density anomalies given with respect to the reference density model.

Here any kind of (anomalous) mass distribution is expressed by its deviation above ( $H(\Omega)>0$ ) and below ( $H(\Omega)<0$ ) a mean sphere at depth  $L$ , i.e. with the radius  $R_L=R-L$  and the density distribution  $\Delta\rho(\Omega)$  (here only lateral variable). Here  $\Omega=(\lambda,\theta)$  denotes the coordinate pair of spherical longitude  $\lambda$  and co-latitude (polar distance)  $\theta$ . In a spherical harmonic approach, the gravitational potential caused by the given mass distribution is given by the spherical harmonic coefficients (e.g. Ramillen 2002, Kuhn and Featherstone 2003a)

$$\bar{V}_{nm} = \mu_n \left( \frac{R_L}{R} \right)^{n+2} \sum_{p=1}^{\infty} \frac{\Gamma(n+3)}{p! \Gamma(n+4-p)} \bar{\kappa}_{p,nm} \quad \text{with the factor} \quad \mu_n = \frac{4\pi GR}{2n+1}$$

and the fully normalized spherical harmonic coefficients  $\bar{\kappa}_{p,nm}$  of the surface density functions

$$\kappa_p(\Omega) = \Delta\rho(\Omega) \frac{H(\Omega)^p}{R_L^{p-1}}, \quad p \in N^+.$$

The disturbing potential of the SEGM is then given by the sum (superposition) of spherical harmonic coefficients for all regarded mass distributions. For the CurtinSEGM, only the first-order approximation ( $p=1$ ) of the above formula has been used (accounts for approx 95 % of the total power). Finally the disturbing potential is converted into geoid heights using Bruns's formula (e.g. Heiskanen and Moritz 1967)

$$\bar{N}_{nm} = \frac{1}{\gamma} \bar{V}_{nm} \quad \text{with the normal gravity } \gamma.$$

For the effect of each mantle layer  $i$  [ $i=1,\dots,115$ ] a set of Love numbers  $G_i$  has been introduced, which accounts for the gravitational effect of a dynamic mantle (e.g. Kogan and McNutt 1993). The corresponding spherical harmonic coefficients are then given by

$$\bar{V}_{p,nm,i} = G_i \mu_n \left( \frac{R_L}{R} \right)^{n+2} \sum_{p=1}^{\infty} \frac{\Gamma(n+3)}{p! \Gamma(n+4-p)} \bar{\kappa}_{p,nm}$$

For the CurtinSEGM, a set of Love numbers has been derived by a simple fit (trial and error) of the disturbing potential (induced by degree 2 – 12) to that of EGM96 (degree 2 – 12). Here only one constant Love number for each layer has been considered instead of one for each spherical harmonic degree (Kogan and McNutt 1993). The set of Love numbers used is illustrated in Figure 4.

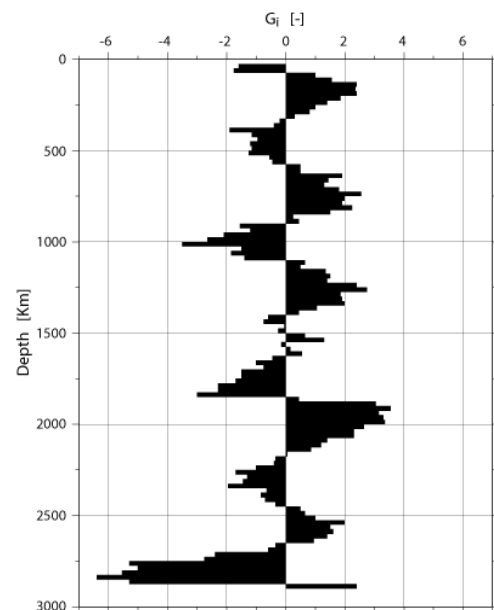


Figure 4: Set of Love numbers used in Curtin SEGM

## Provided Data

All data used to construct the CurtinSEGM are provided by fully normalised spherical harmonic coefficients for the disturbing potential and the corresponding geoid height. The file names of the final spherical harmonic coefficient set as well as the different components are given in Table 4.

**Table 4:** List of all data files combined in CurtinSEGM

Data File	N <sub>max</sub>	Description
<b>JGP95E – Topography/Bathymetry (equivalent rock height)</b>		
erhJGPm05t1_r2670_N360.shc	360	Equivalent rock heights (Topography: $H^{\text{erh}} \geq 0$ )
erhJGPm05t1_r2670_N1440.shc	1440	- " -
erhJGPm05o1_r2670_N360.shc	360	Equivalent rock heights (Bathymetry: $H^{\text{erh}} < 0$ )
erhJGPm05o1_r2670_N1440.shc	1440	- " -
V_erhJGPm05t1_r2670_N360.shc	360	Effect on gravitational potential
V_erhJGPm05t1_r2670_N1440.shc	1440	- " -
V_erhJGPm05o1_r2670_N360.shc	360	- " -
V_erhJGPm05o1_r2670_N1440.shc	1440	- " -
N_erhJGPm05t1_r2670_N360.shc	360	Effect on geoid height
N_erhJGPm05t1_r2670_N1440.shc	1440	- " -
N_erhJGPm05o1_r2670_N360.shc	360	- " -
N_erhJGPm05o1_r2670_N1440.shc	1440	- " -
<b>CRUST 2.0 (crust only)</b>		
V_CurtinSEGMfv1CRUST_N360.shc	360	Anomalous potential of Crust only SEGM
V_CurtinSEGMfv1CRUST_N1440.shc	1440	- " -
N_CurtinSEGMfv1CRUST_N360.shc	360	Geoid height of Crust only SEGM
N_CurtinSEGMfv1CRUST_N1440.shc	1440	- " -
V_CurtinSEGMfv1TopMo_N360.shc	360	Anomalous potential caused by Topo., Bathy. and Moho
V_CurtinSEGMfv1TopMo_N1440.shc	1440	- " -
N_CurtinSEGMfv1TopMo_N360.shc	360	Geoid height caused by Topo., Bathy. and Moho
N_CurtinSEGMfv1TopMo_N1440.shc	1440	- " -
V_CRUST2.0fv1_N180.shc	180	Effect on gravitational potential of crustal mass anomalies
N_CRUST2.0fv1_N180.shc	180	Effect on geoid height of crustal mass anomalies
<b>S12WM13 (mantle only)</b>		
V_S12WM13fv1_N12.shc	12	Effect on gravitational potential of mantle mass anomalies
N_S12WM13fv1_N12.shc	12	Effect on geoid height of mantle mass anomalies
<b>CurtinSEGM Version 1 (Forward Model)</b>		
V_CurtinSEGMfv1_N1440.shc	360	Anomalous potential of CurtinSEGM Version1
V_CurtinSEGMfv1_N1440.shc	1440	- " -
N_CurtinSEGMfv1_N1440.shc	360	Geoid height of CurtinSEGM Version1
N_CurtinSEGMfv1_N1440.shc	1440	- " -

Data can be downloaded free of charge from:

<http://www.cage.curtin.edu.au/~kuhnm/CurtinSEGM/index.html>

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