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## Recent Research Towards an Improved Geoid Model for Australia

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### ABSTRACT

The members of the Western Australian Centre for Geodesy at Curtin University of Technology and our collaborators in Australia and around the world are active in the determination of future generations of the Australian geoid model and its relation to the Australian Height Datum (AHD). As part of ongoing Australian Research Council grant funding, we continue to improve our geoid determination theories, techniques and computer software for provision to the National Mapping Division of Geoscience Australia (formerly AUSLIG). This paper summarises our work over the last year on several of the key aspects to producing a new generation of geoid model for Australia that will better support the direct transformation of GPS-derived ellipsoidal heights to the 1971 realisation of the AHD. These activities include (1) computation of new high-resolution grid of gravimetric terrain corrections from version 2 of the GEODATA 9-arc-second digital elevation model, (2) identification of erroneous ship-track gravity data using multi-mission satellite altimetry, (3) trials of the University of New Brunswick's approach to regional geoid determination over Australia, (4) experiments on gridding two different types of terrestrial gravity anomalies, and (5) investigation of various methods to fit the gravimetric geoid model to the AHD via regional GPS data. Results of experiments on all these facets will be presented, together with some coarse estimates of the improved precision that can be expected from future releases of the Australian geoid model in relation to the AHD.

**KEYWORDS:** geodesy, geoid, gravity, terrain, GPS, height.

## INTRODUCTION

As is well known, the geoid-ellipsoid separation (aka  $N$  value) is required to transform GPS-derived ellipsoidal heights ( $h$ ) to orthometric heights ( $H$ ) using the algebraic expression  $H=h-N$ . Orthometric heights are physically meaningful quantities that are generally compatible with the elevations above local mean sea level as displayed on topographic maps or stored in georeferenced databases.

AUSGeoid98 (Featherstone *et al.*, 2001; <http://www.auslig.gov.au/geodesy/ausgeoid/>) is the latest in the series of Australian geoid models, and thus supersedes all previous releases. Nevertheless, the theories, techniques and data used to compute regional geoid models continue to improve, and it is thus necessary to test these in the Australian context, as well as developing new techniques and theories in Australia. An important aim is to provide an Australian geoid model that will satisfy the needs of a wide-as-possible range of users.

This paper summarises the results of the last year's work by members of the Western Australian Centre for Geodesy at Curtin University of Technology and our collaborators in Australia and around the world towards generating the next generation of the Australian geoid model. This includes:

- (1) Computation of a new high-resolution grid of terrain corrections to terrestrial gravity data from version 2 of the GEODATA 9 arc-second digital elevation model (DEM) of Australia;
- (2) Identification and removal of erroneous ship-track gravity data in the Australian gravity database through comparisons with multi-mission satellite altimetry;
- (3) Trials of the University of New Brunswick's (UNB, Canada) generalised Stokes approach with a deterministically modified integration kernel to regional gravimetric geoid determination over Australia;
- (4) Experiments on gridding two types of terrestrial gravity anomalies to attempt to quantify any long-wavelength errors due to a signal-processing artefact called aliasing; and
- (5) Investigation of the methods to fit the gravimetric geoid model to the Australian Height Datum (AHD) via regional GPS and spirit levelling data tied to the AHD.

As with AUSGeoid98, the new theories, techniques and computer software will be supplied to the National Mapping Division of Geoscience Australia (formerly AUSLIG) for evaluation and the probable release of a new generation of Australian geoid model. This paper will also speculate on the improvements in precision with respect to the AHD (Roelse *et al.*, 1971) that are likely to be made over AUSGeoid98.

## HIGH-RESOLUTION GRAVIMETRIC TERRAIN CORRECTIONS

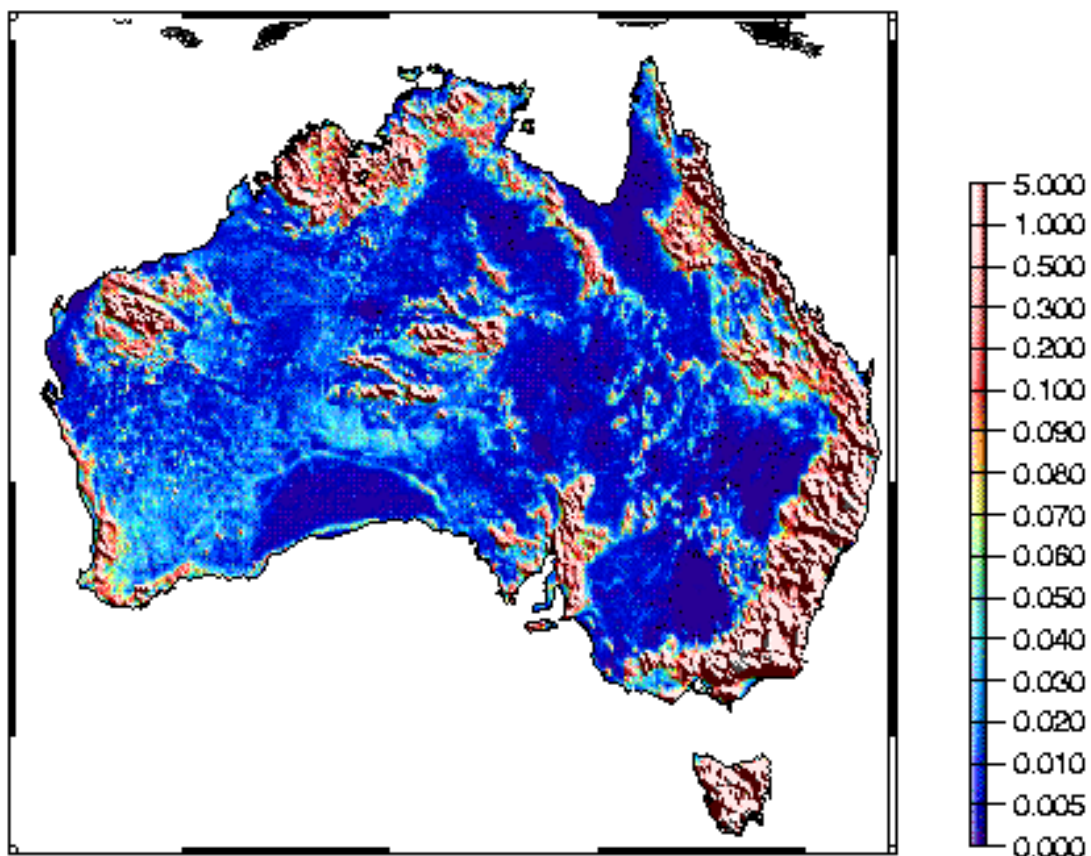
For GPS to be used to its full potential in areas of rugged terrain, the topographic effects on the geoid must be considered. The justification for this is twofold. The mathematical basis of gravimetric geoid determination requires that the effect of topography be taken into account. Secondly, a detailed digital elevation model (DEM) provides short-wavelength contributions to the geoid. Table 1 (taken from Featherstone and Guo, 2001) shows that the inclusion of topographic corrections in AUSGeoid98 significantly improves upon its predecessor AUSGeoid93 (Steed and Holtznagel, 1994), which did not explicitly use such corrections.

AHD height	# points	AUSGeoid93				AUSGeoid98			
		Max	Min	Mean	STD	Max	Min	Mean	STD
>0	1013	3.454	-2.427	0.164	±0.494	3.558	-2.572	-0.002	±0.314
>200	433	2.775	-1.217	0.239	±0.473	1.715	-0.561	0.030	±0.259
>300	304	2.775	-1.217	0.278	±0.481	1.715	-0.561	0.028	±0.264
>400	228	2.775	-0.681	0.313	±0.459	0.984	-0.550	0.020	±0.255
>500	192	2.775	-0.622	0.297	±0.419	0.844	-0.550	0.001	±0.247
>600	139	1.898	-0.799	-0.054	±0.474	0.844	-0.315	-0.003	±0.251

**Table 1.** Descriptive statistics of the differences between GPS-AHD heights and AUSGeoid93 and AUSGeoid98 as a function of increasing AHD height (units in metres)

Kirby and Featherstone (1999) computed the first Australia-wide grid of terrain corrections to gravity data, which was subsequently used in AUSGeoid98 (Featherstone *et al.*, 2001). These terrain corrections used version 1 of the GEODATA 9 arc-second DEM of Australia (Carroll and Morse, 1996) and the planar fast Fourier transform (FFT) implementation of Moritz's (1968) terrain correction algorithm (cf. Schwarz *et al.*, 1990). However, the spatial resolution of the computed terrain corrections had to be reduced to 27 arc-seconds because anomalously large terrain corrections (exceeding 250 mGal) resulted in some areas when the 9 arc-second DEM grid was used. At that time, Kirby and Featherstone (1999) attributed this to the numerical instability in Moritz's (1968) algorithm for densely spaced DEM grids (cf. Martinec *et al.*, 1996).

Kirby and Featherstone (2001) have since discovered that the excessive terrain correction values were caused by anomalously large (up to 76 degrees) horizontal gradients [errors] in version 1 of the GEODATA DEM, and not necessarily by the instability in Moritz's (1968) algorithm (cf. Tsoulis, 2001). Incorrect flow directions of the stream data used to constrain the version 1 DEM caused these spurious gradients (Hutchinson, 2000 pers. comm.). Version 2 of the GEODATA 9 arc-second DEM of Australia was released in 2001 (Hutchinson, 2001; <http://www.auslig.gov.au/products/digidat/dem.htm>), and these anomalous horizontal gradients have now been corrected. This has allowed the full 9 arc-second resolution of the version 2 GEODATA DEM to be used to compute a new high-resolution grid of gravimetric terrain corrections, again based on Moritz's (1968) formula (Figure 1; Kirby and Featherstone, in press).



**Figure 1.** Image of the 9 arc-second gravimetric terrain corrections on land over Australia (Image illuminated from the north-east. Units in mGal. Mercator projection)

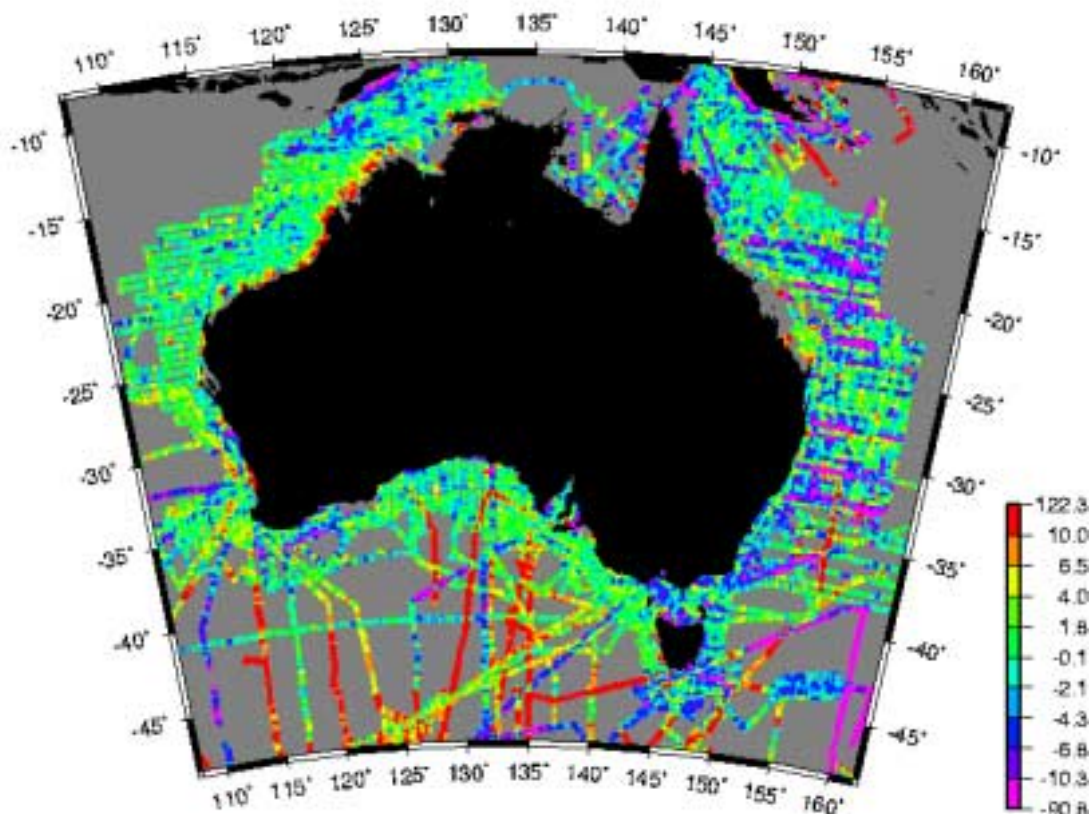
Table 2 shows the descriptive statistics of the version 1 and version 2 GEODATA 9 arc-second DEM grids and the corresponding terrain corrections: the 27 arc-second grid from Kirby and Featherstone (1999); and the 9 arc-second grid from Kirby and Featherstone (in press). Importantly, the newer grid provides higher frequency terrain corrections to gravity data than previously possible over the whole of Australia. It is anticipated that this will improve the new generation of Australian gravimetric geoid model, especially in regions of rugged terrain (cf. Figure 1), as well as improve the computation of mean gravity anomalies for subsequent geoid computation (Featherstone and Kirby, 2000).

	<i>Max</i>	<i>Min</i>	<i>Mean</i>	<i>STD</i>
Version 1, 9 arc-second DEM (metres)	2193.4	-23.8	133.6	±190.2
27 arc-second TCs (mGal)	40.2	0.0	0.1	±0.4
Version 2, 9 arc-second DEM (metres)	2228.0	-16.0	272.0	±191.5
9 arc-second TCs (mGal)	86.9	0.0	0.2	±0.7

**Table 2.** Descriptive statistics of the Australian digital elevation models (DEMs) and corresponding gravimetric terrain corrections (TCs)

## IDENTIFICATION OF ERRONEOUS SHIP-TRACK GRAVITY DATA

AUSGeoid98 included marine gravity data from a combination of satellite altimeter-derived gravity anomalies (Sandwell and Smith, 1997) and the 1998 release of Geoscience Australia's (formerly AGSO) ship-track gravity database. This data combination was achieved using least-squares collocation (e.g., Tscherning *et al.*, 1992), where the altimeter-derived gravity anomalies were 'warped' to fit the ship-track gravity anomalies (Kirby and Forsberg, 1998). This yielded prototype geoid models that were improvements upon those that used ship-track data only, as demonstrated by comparisons with coastal GPS-levelling data (also see Featherstone and Guo, 2001).



**Figure 2.** Differences between GMGA97 multi-mission altimeter-derived gravity anomalies and the 1998 AGSO ship-track gravity anomalies (units in mGal; Lambert projection)

Figure 2 (from Featherstone, submitted1) shows the differences between the GMGA97 multi-mission altimeter-derived gravity anomaly grid (Hwang *et al.*, 1998) and the ship-track gravity observations from the 1998 release of AGSO's ship-track gravity data. Figure 2 shows that large (>10 mGal) differences exist between adjoining and crossing ship-tracks, thus demonstrating the omission of crossover corrections from most of the 1998 AGSO ship-track data (cf. Wessel and Watts, 1988). Unfortunately, the ship-track gravity data are not currently

provided in a format that allows crossover corrections to be applied. Accordingly, it will be necessary to omit ship-track data from future Australian geoid models; at least until crossover corrections can be applied.

Featherstone (submitted1) also shows that there are significant differences ( $>50$  mGal) among the satellite altimeter-derived gravity anomaly grids computed by different groups (e.g., Sandwell and Smith, 1997; Andersen and Knudsen, 1998; Hwang *et al.*, 1998), especially close to the Australian coast. This is due to a combination of the errors induced by incorrectly tracking radar returns from the coastal zone (Deng *et al.*, submitted), poorly modelled shallow-sea tides, and the Gibbs phenomenon (edge-effect) at the coasts in the FFT techniques used.

The above two issues raise the practical problem of choosing which grid of altimeter-derived gravity anomalies to use in future generations of the Australian geoid model. As the Australian ship-track gravity observations are unreliable, they cannot be used to select the most precise grid of altimeter-derived gravity anomalies, neither should they be used in the geoid model. Instead, purely theoretical considerations will probably have to be relied upon to select the altimeter grid. Of the altimeter-derived gravity anomaly grids currently available in the public domain, GMGA97 (Hwang *et al.*, 1998) is recommended purely because it appears to use a geodetically more rigorous theoretical basis than the approximations used by other groups (Hwang, 1998).

## THE UNB TECHNIQUE FOR GRAVIMETRIC GEOID DETERMINATION

The University of New Brunswick's (UNB, Canada) generalised Stokes scheme for regional gravimetric geoid determination (Vaníček and Kleusberg, 1987; Vaníček and Sjöberg, 1991) involves the use of a deterministically modified integration kernel, a degree-20 satellite-only reference field, integration of high-frequency terrestrial gravity anomalies over a spherical cap of 6 arc-degrees radius about each computation point, and a separate computation of the truncation bias using degrees 21-120 from a combined global geopotential model. The formulae for this approach are summarised in Novak *et al.* (2001), together with evidence that the technique can generate a gravimetric geoid model to an accuracy of  $\sim 1$  cm. However, since *ibid* use an error-free synthetic gravity field, the UNB approach has been trailed for Australia used observational data as follows.

The AUSGeoid(UNB) gravimetric geoid model over Australia was computed using exactly the same grid of terrestrial gravity anomalies that was used for AUSGeoid98 (Featherstone *et al.*, 2001). At the time that these computations were undertaken (Featherstone *et al.*, submitted2), the new grid of terrain corrections (described above) was not available. The EGM96S satellite-only global geopotential model (Lemoine *et al.*, 1998), truncated to spherical harmonic degree 20, was used as the reference field. The truncation bias was computed using the 21-120-degree expansion of the EGM96 combined global geopotential model (*ibid*). The primary indirect effect was computed using the quadratic term of Wichiencharoen's (1982) formula, which is consistent with the Moritz (1968) terrain corrections used in the gravity grid.

By way of comparison, an AUSGeoid(RCR) gravimetric geoid model over Australia was computed using the common remove-compute-restore (RCR) approach (e.g., Schwarz *et al.*, 1990). This geoid model also used exactly the same grid of terrestrial gravity anomalies and primary indirect effects as were used for AUSGeoid98. Here, the complete degree-360 expansion of the EGM96 combined global geopotential model was used in conjunction with regional terrestrial gravity data via an unmodified (spherical) Stokes kernel. In order to be consistent with the above application of the UNB technique, the RCR technique was applied over a spherical cap of 6 arc-degrees radius about each computation point. However, the more common application of the RCR technique is to use the whole gravity data rectangle (e.g., Fotopoulos *et al.*, 1999).

<i>Geoid Model</i>	<i>Max</i>	<i>Min</i>	<i>Mean</i>	<i>STD</i>
EGM96S to degree 20	6.970	-6.513	1.530	$\pm 2.505$
EGM96 to degree 360	3.537	-2.441	-0.015	$\pm 0.441$
AUSGeoid98	3.558	-2.572	-0.002	$\pm 0.314$
AUSGeoid(UNB)	3.580	-2.932	0.097	$\pm 0.495$
AUSGeoid(RCR)	5.524	-3.027	0.251	$\pm 0.791$

**Table 3.** Statistics of the absolute differences between the 1013 GPS-AHD data and various geoid models over Australia (units in metres)

Table 3 shows a comparison of various Australian geoid modes with the same nation-wide dataset of 1013 GPS-AHD points that has been used to test AUSGeoid98 (cf. Table 1; Featherstone and Guo, 2001). This shows that AUSGeoid98 provides a better fit to the 1013 GPS-AHD data than the AUSGeoid(UNB) and AUSGeoid(RCR) geoid models. This vindicates the approaches used to compute AUSGeoid98, notably the hybrid approach of a different integration kernel (Featherstone *et al.*, 1998a) with an adaptation of the RCR scheme (Featherstone *et al.*, 2001). However, the comparison in Table 3 is slightly unfair because optimisation of the spherical cap radius was not performed for the AUSGeoid(UNB) and AUSGeoid(RCR) geoid models, whereas it was for AUSGeoid98 (*ibid*).

From the results summarised in Table 3, the AUSGeoid(UNB) and AUSGeoid(RCR) geoid models produce a worse fit to the GPS-AHD data than the degree-360 expansion of EGM96. This is consistent with the observations of Featherstone *et al.* (2001), Forsberg and Featherstone (1998) and Forsberg (1998a), where larger spherical cap radii yield worse fits than smaller cap radii (cf. Kearsley, 1988). This is attributed to peculiarities in the Australian gravity data. Table 3 also shows that the AUSGeoid(UNB) geoid model is superior to the AUSGeoid(RCR) geoid model for the transformation of GPS-derived ellipsoidal heights to the AHD. However, further work with the new gravity data (i.e., with the high-resolution terrain corrections and satellite altimeter data; both described earlier) is required to confirm this observation.

## GRIDDING DIFFERENT TYPES OF TERRESTRIAL GRAVITY ANOMALIES

The fast Fourier transform (FFT) technique, which is necessary for the practical computation of a high-resolution geoid model over an area the size of Australia, requires a regular geographic grid of mean gravity anomalies (e.g., Schwarz *et al.*, 1990). In order to predict gravity on a regular grid from irregularly spaced gravity observations requires interpolation, and sometimes extrapolation. As is well known from signal sampling theory, the interpolation process is subject to a phenomenon similar to aliasing in the presence of improperly sampled high frequencies. If there are insufficient observations to sample the whole gravity field spectrum, then high-frequency signals are incorrectly sampled (i.e., pseudo-aliased) into the low frequencies. This generates spurious long-wavelength gravity anomalies that propagate into the computed geoid model, which is especially problematic because most of the geoid's power is contained in the low frequencies.

For AUSGeoid98, simple Bouguer gravity anomalies were used during the gridding process (Featherstone *et al.*, 2001). However, the simple Bouguer correction to gravity data can omit the high frequency effects by way of the gravimetric terrain correction (described earlier) and is thus subject to the effects of pseudo-aliasing. The justification for not using refined Bouguer anomalies (i.e., simple Bouguer anomaly plus the terrain correction) in AUSGeoid98 is that it prevents the correct computation of mean gravity anomalies (Featherstone and Kirby, 2000). Featherstone *et al.* (2001) acknowledge the possible limitation of not using refined Bouguer anomalies. Therefore, recent experiments (Goos *et al.*, submitted) have focussed on using refined Bouguer anomalies during the gridding process and determining the effect that this has on the computed regional geoid model.

The two new prototype geoid models, AUSGeoid(SBA) and AUSGeoid(RBA) shown in Table 4, have been computed using the RCR technique (described earlier) applied over the whole gravity data area, because this approach is most sensitive to low-frequency errors in terrestrial gravity data (cf. Vaniček and Featherstone, 1998). Also, it is necessary to point out that these two prototype geoid models have been computed from the 2001 release of Geoscience Australia's land gravity database (recall that the ship-track gravity data are unreliable), the GMGA97 satellite altimeter-derived marine gravity anomalies, and the new high-resolution grid of terrain corrections from the version 2 GEODATA 9 arc-second DEM (described earlier).

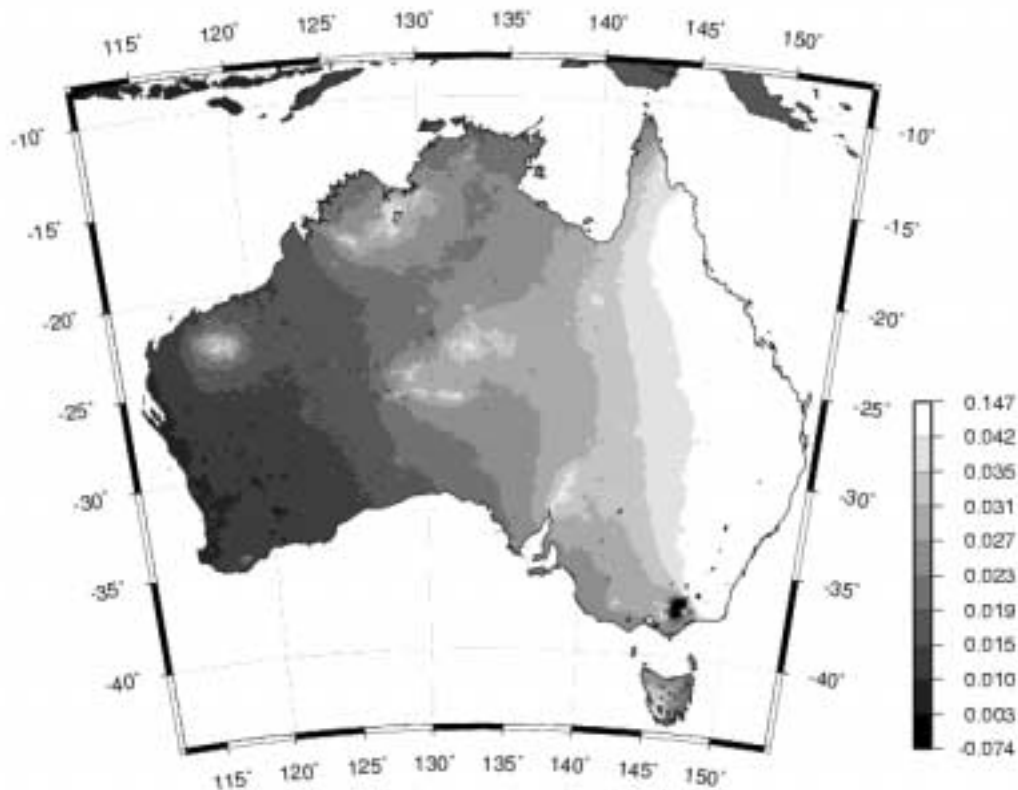
<i>Geoid Model</i>	<i>Max</i>	<i>Min</i>	<i>Mean</i>	<i>STD</i>
AUSGeoid98 (i.e., simple Bouguer gridding)	3.558	-2.572	-0.002	±0.314
AUSGeoid(SBA) (simple Bouguer gridding)	5.169	-1.691	1.449	±0.741
AUSGeoid(RBA) (complete Bouguer gridding)	5.222	-1.657	1.487	±0.749

**Table 4.** Statistics of the differences between the 1013 GPS-AHD data and two prototype geoid models over Australia as a function of gravity gridding technique (units in metres)

Simply by comparing the standard deviation shown in row 6 of Table 3 with that shown in row 3 of Table 4 demonstrates that an improvement of a few centimetres in the fit of new generations of Australian gravimetric geoid to the GPS-AHD data can be expected from the use of the new data. However, recall that the geoid model in Table 3 uses a 6-degree spherical cap radius, whereas the geoid models in Table 4 use the entire (rectangular) gravity data area. Therefore, the above estimate of the likely improvement is very approximate, and will probably be larger when the integration domain is optimised (cf. Featherstone *et al.*, 2001).

Table 4 shows that the reduction in pseudo-aliasing offered by the use of refined, as opposed to simple, Bouguer gravity anomalies in the gravity gridding stage is insignificant for Australia. Therefore, relying upon the arguments in Featherstone and Kirby (2000) of the more accurate computation of mean Faye gravity anomalies, this appears a better approach for Australia versus the attempted reduction of pseudo-aliasing by the application of terrain corrections before, as opposed to after, gravity anomaly gridding.

Figure 3 shows a map of the differences between the AUSGeoid(SBA) and AUSGeoid(RBA) geoid models, which is correlated with the Australian terrain corrections (cf. Figure 1) and hence the topography. These differences come mainly from the difference between mean values of the terrain corrections when applied to each gravity observation (the RBA model) and the mean terrain corrections applied over a whole compartment, as per Featherstone and Kirby (2000) (the SBA model). Acknowledging the limitations of the GPS-AHD data, the small (and statistically insignificant) improvement in Table 4 could be attributed to the more appropriate use of the SBA model in Australia.



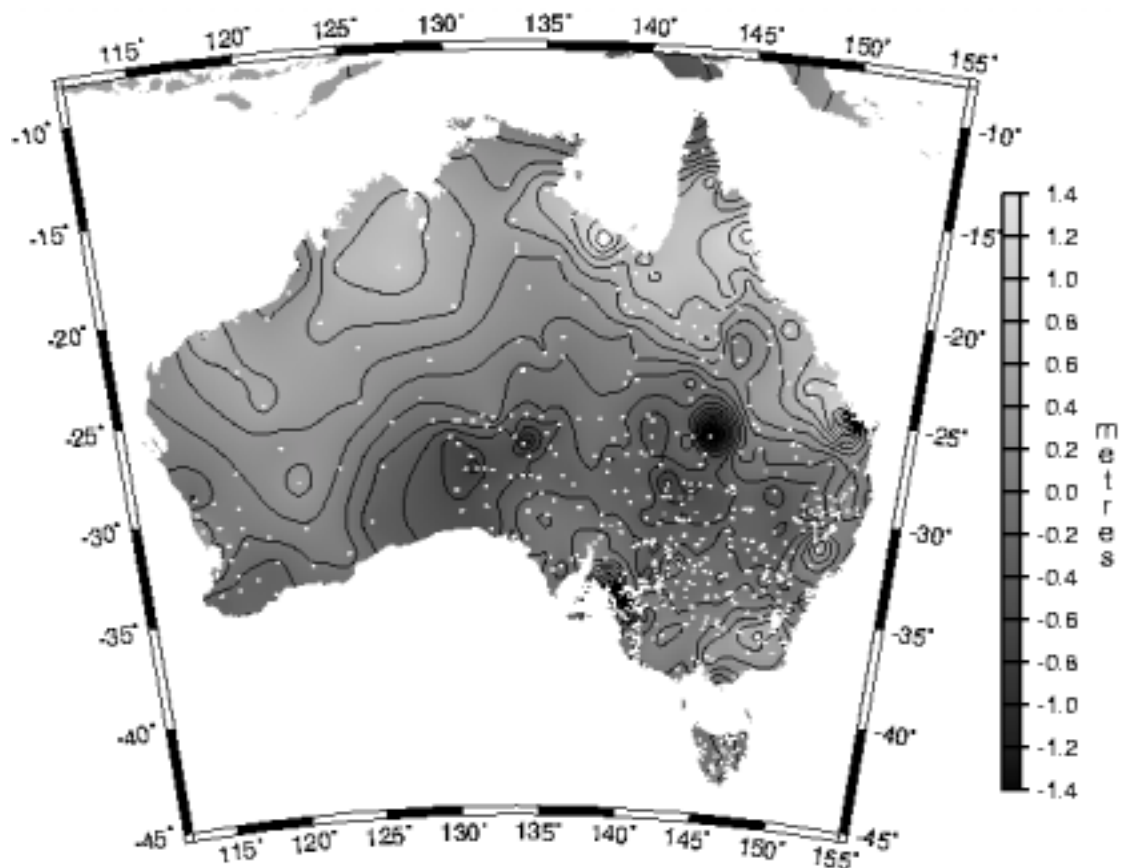
**Figure 3.** Differences between the AUSGeoid(SBA) and AUSGeoid(RBA) geoid models (units in metres).

## FITTING THE GRAVIMETRIC GEOID MODEL TO THE AHD VIA GPS DATA

A key question in Australian geoid-related research is the explanation and modelling of the differences between the AHD and AUSGeoid98 (cf. Figure 4 and Table 1; also see Featherstone 1998; Featherstone *et al.*, 2001). Of most practical relevance in Australia is the provision of 'geoid-type' models (i.e., those that do not necessarily represent the equipotential geoid) for the **direct** transformation of GPS-derived ellipsoidal heights to the AHD and *vice versa*. The need for this type of model is based upon the arguments presented in Featherstone (1998).

In summary, the provision of a 'geoid-type' model that is a best fit to the AHD avoids the users of our 'product' to post-process GPS-AUSGeoid98-derived heights to make them compatible with existing benchmarks on the AHD (Featherstone *et al.*, 1998b). With the ready availability of precise long-baseline GPS positioning techniques, such as AUSPOS (<http://www.auslig.gov.au/geodesy/sgc/wwwgps/>), the differences between AUSGeoid98 and the AHD are becoming more apparent (e.g., Featherstone and Dent, in press). This is because the absolute geoid heights are used, as opposed to the previous application of geoid height differences, where correlated errors would cancel (e.g., Kearsley, 1988).

At present, the most promising approach to the solution of these differences is the *combination* of a gravimetric-only geoid model with GPS and AHD data (cf. Featherstone, 2000). This combined approach is commonly used in many other parts of the world, such as the USA (Smith and Roman, 2001). Given the current status of gravimetric geoid modelling in Australia, it is now becoming necessary to account for the distortions in the AHD (e.g., Featherstone, 1998), thus providing a more direct transformation of GPS-derived heights in Australia. This approach is being investigated with a view to providing a model of the separation between the GRS80 ellipsoid and the AHD, as opposed to the separation between the GRS80 ellipsoid and the equipotential geoid.



**Figure 4.** Image of the differences between 1013 GPS-AHD heights (white circles) and AUSGeoid98 [Lambert projection from GRS80]

However, it is important to acknowledge that the 1013 GPS-levelling data used by Featherstone and Guo (2001) and in the earlier parts of this study have only recently been discovered less reliable than first thought (Johnston and Luton, 2001). In addition to the distortions in the AHD (e.g., Featherstone, 1998), these GPS data are of a variety of vintages, with many being collected using old instruments and processed using sub-optimal techniques (e.g., without precise ephemerides). Therefore, simple comparisons of gravimetric geoid models with these GPS-AHD data, while useful because they are the only [partly] independent data with which to test gravimetric geoid models on land, remain equivocal because of the error budget in the GPS-AHD data.

While acknowledging the above limitation, five prototype combined (i.e., AUSGeoid-GPS-AHD) 'geoid-type' models have been constructed so as to give some indication of the improvement that can be made to the gravimetric-only geoid model of Australia (cf. Featherstone, 2000). Several mathematical models have been

trailed in this regard, including a simple bias, a bias and a tilt, and higher order polynomials. Table 5 shows the statistics of the differences between the geoid-type models and the GPS-AHD data, before and after the various ‘corrector’ surfaces have been fit to the residuals between the gravimetric-only AUSGeoid98 and the GPS-AHD data. A subset of the 1013 GPS-AHD points described earlier was used for the analysis, which included 953 points on the Australian mainland (i.e., excluding Tasmania), after the removal of suspect values with differences greater than 3RMS between AUSGeoid98 and the GPS-AHD data.

<i>Model</i>	<i>Max</i>	<i>Min</i>	<i>Mean</i>	<i>STD</i>
AUSGeoid98 (no surface fit)	0.933	-0.759	0.013	± 0.258
AUSGeoid98(B) (with bias only)	0.920	-0.772	0.000	± 0.258
AUSGeoid98(BT) (with bias and tilt surface fit)	0.620	-1.180	0.000	± 0.194
AUSGeoid98(P1) (with first order polynomial fit)	0.623	-1.038	0.000	± 0.199
AUSGeoid98(P2) (with second order polynomial fit)	0.625	-1.064	0.000	± 0.176
AUSGeoid98(P3) (with third order polynomial fit)	0.537	-0.734	0.000	± 0.162

**Table 5.** Statistics of the differences between the 953 GPS-AHD data and various combined ‘geoid-type’ models over the Australian mainland (units in metres).

The first row in Table 5 gives the statistics for the differences between AUSGeoid98 and the 953 GPS-AHD data, showing an agreement of approximately 26 cm over the Australian mainland (after outlier detection). By applying a simple four-parameter fit, representing a bias and tilt (Table 5), the agreement between the two data sets improves to less than 20 cm. Additional surface fits were tested which basically included a multiple regression equation polynomial fit of various orders and basis functions (i.e., trigonometric, non-trigonometric and mixed).

It is shown by the first, second and third order polynomial fits (Table 5) that as the order increases, and consequently the number of parameters defining the ‘corrector’ surface increases, the standard deviation drops from approximately 20 cm to 16 cm. Higher order polynomial fits were also tested up to sixth order, which resulted in a standard deviation of fit as low as 14 cm. However, the apparent improvement in fit comes at the cost of numerical stability. As the order of the polynomial and the number of parameters increases, the condition number of the solution also increases dramatically resulting in an unstable solution. An additional consideration concerning the use of biases, tilts and polynomials is that they are not constrained in areas with no GPS-AHD data, notably offshore.

Least-squares collocation can also be applied as shown in Forsberg (1998b) and Featherstone (2000), whereby the majority of the trend is removed first, then the remaining residuals are modelled by a ‘corrector’ surface generated using least-squares collocation. This approach has not yet been tested fully over Australia. However, it is not directly comparable with the results shown in Table 5 as it entails using one of the above ‘corrector’ surface fits, followed by an additional modelling of the remaining residuals through least-squares collocation.

It is evident from these preliminary tests that fitting the gravimetric geoid model to the AHD via GPS data requires that a balance be found between the desired level of fit and numerical stability of the solution (Fotopoulos *et al.*, submitted). Further studies must also be conducted on using a reliable independent test data set, notably improved GPS data (cf. Johnston and Luton, 2001), and statistical tests for evaluating the performance of the different combined geoid-type models.

## CONCLUDING REMARKS

This paper has presented the results of research work conducted over the last year by members of the Western Australian Centre for Geodesy at Curtin University of Technology and some of our collaborators on the production of a new generation of Australian geoid model and its relation to the AHD. This has included the computation of higher resolution gravimetric terrain corrections based on the corrected version 2 GEODATA 9 arc-second DEM of Australia, identification and removal of erroneous ship-track gravity data around Australia, trials of the University of New Brunswick’s approach to regional geoid determination, experiments on the role of

terrain corrections in the computation of a regular grid of terrestrial gravity anomalies, and fitting the gravimetric geoid model to GPS-AHD data in order to yield a surface that is better suited to the **direct** determination of AHD heights from GPS.

Since several of these studies have been conducted in parallel, the results from each have not been applied in sequence to the subsequent tests. Accordingly, no definite conclusions as to the precision of any new geoid model should be reached at this stage. However, simple inspection of the standard deviations of the fit of the numerous prototype Australian geoid models to the GPS-AHD data do indicate that some improvements have been made, notably through the 'corrector' surface fitting (which is to be expected). However, this claim must be balanced against the suspect quality of the GPS-AHD data used in the tests. Therefore, in a subsequent study, a sequential approach based on the findings presented here will be conducted to ascertain the optimal approach, including some empirical optimisation of the integration domain.

Nevertheless, some preliminary conclusions can be made from the current studies. Specifically, a gravimetric-only geoid model of Australia is not likely to make a huge improvement upon AUSGeoid98 for the transformation of GPS-derived ellipsoidal heights to the AHD. Therefore, it will be necessary to use the combined methods, probably via least squares collocation, to produce a combined geoid-type model. Subject to the outcome of discussions with the National Mapping Division of Geoscience Australia (formerly AUSLIG), a new gravimetric-only or geoid-type model may be released some time in the near future, say as AUSGeoid2002.

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